

Problem 1. *The function*

$$H = f(t) = 2000 + 1200 \cos(t/10), \quad 0 \leq t \leq 30$$

gives the height (in feet) of a balloon at time t minutes past noon.

(a) At what average rate (with respect to time) does the balloon ascend over each of the following time intervals:

i. $t \in [10, 10.1]$

ii. $t \in [10, 10.01]$

iii. $t \in [10, 10.001]$

iv. $t \in [9.9, 10]$

v. $t \in [9.99, 10]$

vi. $t \in [9.999, 10]$

*(b) What can you say about the balloon's **instantaneous** rate of ascent at 12:10pm?*

(c) Express the approximate change in height ΔH of the balloon between 12:10pm and Δt minutes past 12:10pm in terms of Δt . Assume $\Delta t \sim 0$.

(d) Use part c) to estimate when the balloon is 20 feet lower than it is at 12:10pm.

Solution: (a) Let

$$\Delta H = f(10 + \Delta t) - f(10)$$

be the change in the balloon's height between 12:10pm and Δt minutes past 12:10pm. The table below shows the balloon's average rate of ascent, $\Delta H/\Delta t$, measured in ft/min, for the required six values of Δt .

Δt (sec)	$\Delta H/\Delta t$ (ft/min)
-0.1	-100.651
-0.01	-100.944
-0.001	-100.973
0	-
0.001	-100.980
0.01	-101.009
0.1	-101.299

(b) The above calculations suggest that at 12:10pm the balloon is descending at a rate between 100.973 ft/min and 100.980 ft/min. Our best estimate would be that the instantaneous rate of *ascent* at 12:10pm is approximately

$$\frac{1}{2}(-100.973 + -100.980) \text{ ft/sec} \sim -100.977 \text{ ft/min.}$$

(c) From part b), we know that for $\Delta t \sim 0$,

$$\frac{\Delta H}{\Delta t} \sim -100.977$$

so

$$\Delta H \sim -100.977 \Delta t.$$

(d) The balloon is 20 feet lower than it is at 12:10pm when $\Delta H = -20$. Then since

$$\Delta H \sim -100.977 \Delta t,$$

$$-20 \sim -100.977 \Delta t,$$

and

$$\Delta t \sim -20 / -100.977 \sim 0.2.$$

So the balloon is 20 feet lower than it is at 12:10pm at about 0.2 minutes (12 seconds) past 12:10pm.